NTNU | Norwegian University of Science and Technology

CRYPTO API FAILURES

TTM4205 – Lecture 16

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20.10.2025

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Reference Group Meeting

We had a reference group meeting today and will publish the notes soon.

We will also open for presentations of the technical essay on Nov 10th.

We will organize a new ChipWhisperer setup session on Friday this week.

We will offer additional exercise/lab sessions on Wednesdays from Nov 5th.

Technical Essay

This assignment is to write a technical essay and give a presentation about a scientific topic related to the content given in the course description: either a topic not covered by the lectures or a topic from the lectures more in-depth.

It will be joint work in groups of two or three, and the essay should be roughly 8 to 10 pages long, in addition to references. The topic, scope, and group must be approved by the staff (through dialog over email).

Technical Essay

All essays and presentation slides must be written in LaTeX,
and we provide mandatory templates to be used at:

- https://www.overleaf.com/read/nhcnrbnwzmcw (essay) and,
- https://www.overleaf.com/read/zjqxggmjnzqp (presentation).

Technical Essay

This assignment counts for at most 40 points, based on the following criteria: scientific correctness, quality of writing, the structure of the essay, presentation (figures/tables), referencing, relevant and consistent background material, clear and detailed main section(s), and justification of conclusions.

Technical Essay

Important dates and tasks:

- Topic/scope/group approval (mandatory, email): **October 31st**
- Short oral presentations (mandatory): **November 10th** or **14th** or **17th**
- Draft submission for feedback (voluntary): **November 21st**
- Receive feedback on draft (voluntary): **December 5th**
- Final submission (mandatory): December 19th at 23:59.

All assignments must be handed in at `https://ovsys.iik.ntnu.no`.

Technical Essay

We suggest the following topics, but you can also choose your own:

- ~~Cryptographic Fuzzing and Static Analysis~~
- ~~Formally Verified Cryptographic Code~~
- ~~Vulnerabilities in Threshold Signatures~~
- Degenerate Edwards Curve Attacks
- SCA Against Post-Quantum Cryptography
- ~~More Advanced SCA with ChipWhisperer~~

If choosing your own, you are expected to provide a (preliminary) title and scope, in addition to at least two (academic) references.

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Reference Material

These slides are based on:

- The referred papers in the slides
- JPA: parts of chapter 9 to 12
- DW: parts of chapter 5 to 7

Protocol APIs

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By this we mean, on a high level, a server that:

- holds secrets where clients can make queries
- holds secrets that clients can interact with
- combine inputs to verify batches at once

Protocol APIs

We will look at examples where a client can:

- extract secret signing keys
- forge signatures
- trick a verifier

Several of which are similar to the weekly problems.

We will also look at some mitigations to these issues.

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Recap: Schnorr Signatures

Let $\mathbb{G}$ be a group of prime order p and let g be a generator for $\mathbb{G}$. Denote by pp the public parameters $(\mathbb{G}, g, p)$.

Let H be a cryptographic hash function that outputs uniformly random elements in $\mathbb{Z}_p$.

Let the secret key $\mathsf{sk} \leftarrow\!\!\$\ \mathbb{Z}_p$ be sampled uniformly at random, and let the public key be $\mathsf{pk} = g^{\mathsf{sk}}$, where pk is made public.

Recap: Schnorr Signatures

The Schnorr signature of message m is computed as:

Sample random $r \leftarrow\!\!\$\ \mathbb{Z}_p$ and compute $R = g^r$.

Compute the output challenge as $c = H(\mathsf{pp}, \mathsf{pk}, m, R)$.

Compute the response $z = r - c \cdot \mathsf{sk}$. Output $\sigma = (c, z)$.

To verify the signature, compute $R' = g^z \cdot \mathsf{pk}^c$ and check if $c \stackrel{?}{=} H(\mathsf{pp}, \mathsf{pk}, m, R')$. If correct, accept, and otherwise reject.

Distributed Deterministic Schnorr Signatures

Assume that two parties P_0 and P_1 wants to compute a joint Schnorr signature. Then P_i does the following:

KGen :

- Sample random $\mathsf{sk}_i \leftarrow\$ \mathbb{Z}_p$ and compute $\mathsf{pk}_i = g^{\mathsf{sk}_i}$.
- Send pk_i to party P_{1-i}. Set $\mathsf{pk} = \mathsf{pk}_0 \cdot \mathsf{pk}_1 = g^{\mathsf{sk}_0 + \mathsf{sk}_1}$.

This is called an additive secret sharing of the signing key.

Question: How can a malicious client P_0 interacting with an honest (protocol API) P_1 break this key generation?

Distributed Deterministic Schnorr Signatures

Sign:

- Compute $r_i = \mathsf{H}(sk_i, m)$ deterministically and then $R_i = g^{r_i}$.
- Send R_i to party P_{1-i}. Set $c = H(\mathsf{pp}, \mathsf{pk}, m, R_0 \cdot R_1)$.
- Send the response $z_i = r_i - c \cdot \mathsf{sk}_i$ to party P_{1-i}.

The signature $\sigma = (c, z_0 + z_1)$ can be verified as usual.

Question: How can a malicious client P_0 interacting with an honest (protocol API) P_1 break this signature scheme?

Attacks

- P_0 can wait, then sample sk, and set $\mathsf{pk}_0 = g^{\mathsf{sk}} \cdot (pk_1)^{-1}$ s.t. $\mathsf{pk}_0 \cdot \mathsf{pk}_1 = g^{\mathsf{sk}}$.
  - This way, P_0 controls the key and can create signatures fully on their own.
- P_0 can force nonce re-use in consecutive rounds s.t. $m = m'$ but $c \neq c'$.
  - P_1 will set $r_1 = \mathsf{H}(sk_1, m)$ both times, but P_0 can set R_0 and R_0' arbitrarily.
  - Then $c = H(\mathsf{pp}, \mathsf{pk}, m, R_0 \cdot R_1)$ will be unequal to $c' = H(\mathsf{pp}, \mathsf{pk}, m, R_0' \cdot R_1)$
  - P_1 outputs distinct responses $z_1 = r_1 - c \cdot \mathsf{sk}_1$ and $z_1' = r_1 - c' \cdot \mathsf{sk}_1$.
  - We can extract sk_1 by computing $(z_1 - z_1')/(c' - c)$. P_0 can sign on their own.

Avenues for Attacks

- The adversary can control the key generation
- The adversary can control the nonce products
- (The adversary can abort to deny signatures)
- (All parties need to be online to sign together)

Mitigations in Practice

- Send hashes in an extra round in KGen and keep state in Sign
- This ensures that keygen is honest and correct nonces are formed
- Make it a t-out-of-n signature instead of 2-out-of-2 for robustness

Proactive Two-Party Signatures for User Authentication

Antonio Nicolosi, Maxwell Krohn, Yevgeniy Dodis, and David Mazières
NYU Department of Computer Science
{nicolosi,max,dodis,dm}@cs.nyu.edu

Figure: https://www.scs.stanford.edu/~dm/home/papers/nicolosi:2schnorr.pdf

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BLS Signatures

Let $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ be groups of prime order p with generators g_1, g_2, g_T. Let $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ be a bilinear paring s.t. $\hat{e}(g_1^a, g_2^b) = g_T^{a \cdot b}$ for all $a, b \in \mathbb{Z}_p$.

Let H be a hash function that outputs uniformly random elements in $\mathbb{G}_2$.

Let the secret key $\mathsf{sk} \leftarrow\!\!\$\ \mathbb{Z}_p$ be sampled uniformly at random, and let the public key be $\mathsf{pk} = g_1^{\mathsf{sk}}$, where pk is made public.

A signature is computed as $\sigma = H(m)^{\mathsf{sk}}$. The verifier checks whether the element $\hat{e}(g_1, \sigma)$ equals $\hat{e}(\mathsf{pk}, H(m))$. If correct; accept, otherwise reject.

BLS Multi-Signatures

We can efficiently verify many signatures at once:

- Given many triples $(\mathsf{pk}_i, m_i, \sigma_i)$, compute: $\sigma = \Pi_i \sigma_i$
- Verify all signatures as: $\hat{e}(g_1, \sigma) = \Pi_i \hat{e}(\mathsf{pk}_i, H(m_i))$
- If all messages are identical: $\hat{e}(g_1, \sigma) = \hat{e}(\Pi_i \mathsf{pk}_i, H(m))$
- If the same signers we can aggregate keys: $\mathsf{apk} = \Pi_i \mathsf{pk}_i$

Question: Fix m and pk_0, how can an adversary forge a signature for pk_0 that verifies in the aggregated setting?

Potential Attacks

- Set $\mathsf{pk}_1 = g_1^{\alpha} \cdot (\mathsf{pk}_0)^{-1}$ and signature $\sigma = H(m)^{\alpha}$
- Then $\hat{e}(g_1, \sigma) = \hat{e}(g_1^{\alpha}, H(m)) = \hat{e}(\mathsf{pk}_0 \cdot \mathsf{pk}_1, H(m))$

Mitigations in Practice

- Require a proof for secret key knowledge
- Only aggregate distinct messages each time
- Verify a random linear combination of keys/signatures

Compact Multi-Signatures for Smaller Blockchains

Dan Boneh[1], Manu Drijvers[2,3], and Gregory Neven[2]

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[3] ETH Zurich

Figure: https://eprint.iacr.org/2018/483.pdf

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DL Parameters

For security of (EC)DH and (EC)DSA, we need to work in prime order (sub-) groups for the discrete logarithm problem to be hard. What happens if this is not the case?

DL Attacks

Recall from earlier that:

- Hardness of DL depends on the divisors p of the order n
- We have generic attacks that runs in $\sqrt{p}$ time
- We have sub-exponential attacks for finite field groups

Faulty parameters

Question: What information might leak if:

- The order of the (sub-) group is not prime?
- The element is not in the correct (sub-) group?

Use $g^{sk} \mod p$ as an example (EC in weekly problems).

Question: How might this happen in practice?

Faulty parameters

- If it is not prime, we can solve the problem in prime subgroups
- If it is not in the correct subgroup, DL might be easy there
- If we can do this repeatedly, we can find the DL using CRT

Mitigations in Practice

Always verify:

- given parameters
- input elements
- output elements

Measuring small subgroup attacks against Diffie-Hellman

Luke Valenta*, David Adrian†, Antonio Sanso‡, Shaanan Cohney*,
Joshua Fried*, Marcella Hastings*, J. Alex Halderman†, Nadia Heninger*
*University of Pennsylvania
†University of Michigan
‡Adobe

Figure: https://eprint.iacr.org/2016/995.pdf

In search of CurveSwap: Measuring elliptic curve implementations in the wild

Luke Valenta*, Nick Sullivan†, Antonio Sanso‡, Nadia Heninger*
*University of Pennsylvania, †Cloudflare, Inc., ‡Adobe Systems

Figure: https://eprint.iacr.org/2018/298.pdf

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The API must always:

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- verify protocol parameters

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- verify protocol parameters
- verify API inputs

The API must always:

- verify protocol parameters
- verify API inputs
- check API outputs

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The API must always:

- verify protocol parameters
- verify API inputs
- check API outputs
- enforce honest interaction
- avoid corner case leakage
- hinder replay attacks

Questions?